Austen Mazenko

Characterization of Certain Non-Homogeneous Meta-Fibonacci Sequences

The Fibonacci recursion is one of the most beautiful recursive sequences in mathematics, possessing countless patterns and applications, but related meta-Fibonacci sequences are known for their chaotic behaviors and sensitivity to initial conditions. In this research, we considered the effects on the structure and order of meta-Fibonacci sequences which had non-homogeneous terms in their definitions. Java 7 was utilized to generate sequences with given recursions, which were then analyzed for patterns both graphically and through other methods such as finding successive differences of terms as well as asymptotics. General trends observed in such sequences, such as the eventual periodic nature exhibited by those with particular non-homogeneous terms, were noted and subsequently proven through mathematical methods. Additionally, drawing off of the observation that a majority of meta-Fibonacci sequences which have been thoroughly studied and found to exhibit sorts of patterns are slow, which is to say each term is either 0 or 1 greater than the previous term, we described an infinite family of slow meta-Fibonacci sequences with slow non-homogeneous terms. Using mathematical induction, the infinite family was proven to be, in fact, infinite, and patterns of its structure were described. This work opens up a new type of meta-Fibonacci sequence in non-homogeneous ones with not necessarily constant terms, and has possible applications both in chaos theory, when treated as discrete dynamical systems, and communications theory with regards to canceling out noise.